(1)

a)

x	$-\infty$		-2		1		3	$+\infty$
$\lvert x-3\rvert$	+	+	+	+	+	0	+	
$(x+2)(x-1)$	+	0	−	0	+	+	+	
T	+	SS	−	SS	+	0	+	

$A =]-2, 1[\cup \{3\}$

$|x-3| = 0 \Leftrightarrow x = 3$

$(x+2)(x-1) = 0 \Leftrightarrow x+2 = 0 \vee x-1 = 0 \Leftrightarrow x = -2 \vee x = 1$

b) $int(B) =]-\infty, 2[$

$fr(B) = \{2, 4\}$

$B' =]-\infty, 2]$

$A \cup B =]-\infty, 2] \cup \{3, 4\}$

$\overline{A \cup B} =]-\infty, 2] \cup \{3, 4\}$

c) $\overline{B} =]-\infty, 2] \cup \{4\} = B$, logo é fechado

$B \neq int(B)$, logo não é aberto

(2)

a) A:

x	$-\infty$		-2		-1		1		2	$+\infty$
$\log(x^2)$	+	+	+	0	−	0	+	+	+	
$\lvert x^2-4\rvert$	+	0	+	+	+	+	+	0	+	
T	+	SS	+	0	−	0	+	SS	+	

$A =]-\infty, -1] \cup [1, +\infty[\setminus \{-2, 2\}$

$\log(x^2) = 0 \Leftrightarrow x^2 = 1 \wedge x^2 > 0 \Leftrightarrow (x = -1 \vee x = 1) \wedge x \neq 0 \Leftrightarrow x = -1 \vee x = 1$

$|x^2-4| = 0 \Leftrightarrow x^2 - 4 = 0 \Leftrightarrow x^2 = 4 \Leftrightarrow x = -2 \vee x = 2$

$B: |x^2-1| = 1 \Leftrightarrow x^2 - 1 = -1 \vee x^2 - 1 = 1 \Leftrightarrow x^2 = 0 \vee x^2 = 2 \Leftrightarrow$

$\Leftrightarrow x = 0 \vee x = -\sqrt{2} \vee x = \sqrt{2}$

$B =]-\sqrt{2}, 0[\cup]0, \sqrt{2}[$

b) $A \cup B = \mathbb{R} \setminus \{-2, 0, 2\}$

$int(A \cup B) =]-\infty, -2[\cup]-2, 0[\cup]0, 2[\cup]2, +\infty[$

$A \cap B =]-\sqrt{2}, -1] \cup [1, \sqrt{2}[$

$min(A \cap B) =]-\infty, -\sqrt{2}]$

$B' = [-\sqrt{2}, 0] \cup [0, \sqrt{2}]$

③

a) $int(A) =]-\pi, -1[\cup]e, 10[$

$fr(A) = \{-\pi, -1, 0, e, 10\}$

$A' = [-\pi, -1] \cup [e, 10]$

b) $\overline{A} = [-\pi, -1] \cup [e, 10] \cup \{0\} = A$, logo A é fechado

$int(A) \neq A$, logo A não é aberto

c) $B: x = \begin{cases} -\frac{7}{m+1} & \text{se } m \text{ ímpar} \\ \frac{7}{m+1} & \text{se } m \text{ par} \end{cases} \rightarrow \{-\frac{7}{2}, \frac{7}{3}, -\frac{7}{4}, \frac{7}{5}, -\frac{7}{6}, 1, -\frac{7}{8}, \frac{7}{9}, \dots, 0\}$

$fr(A \cup B) = \{-\pi, -1, 0, e, 10\} \cup B \setminus A$

$(A \cup B)' = [-\pi, -1] \cup [e, 10]$ $\qquad \overline{A \cup B} = [-\pi, -1] \cup [e, 10) \cup \{0\} \cup B \setminus A$

d) minorantes de $A \cup B$: $x =]-\infty, -\frac{7}{2}]$

majorantes de $A \cup B$: $x = [10, +\infty[$

e) supremo de $A \cup B$: $x = 10$ $\qquad$ máximo de $A \cup B$: $x = 10$

ínfimo de $A \cup B$: $x = -\frac{7}{2}$ $\qquad$ mínimo de $A \cup B$: $x = -\frac{7}{2}$

④

a) A:

x	$-\infty$		$-\sqrt{2}$		1		$\sqrt{2}$		2	$+\infty$
$2-\lvert -4x+6 \rvert$	$-$	$-$	$-$	0	$+$	$+$	$+$	0	$-$	
$\log_2^2(x^2-1)$	$+$	0	$+$	$+$	$+$	0	$+$	$\nexists$	$+$	
T	$-$	SS	$-$	0	$+$	SS	$+$	0	$-$	

$A = [1, \sqrt{2}[\cup]\sqrt{2}, 2]$

$2 - |-4x+6| = 0 \Leftrightarrow |-4x+6| = 2 \Leftrightarrow -4x+6 = -2 \vee -4x+6 = 2 \Leftrightarrow x = 2 \vee x = 1$

$\log_2^2(x^2-1) = 0 \wedge x^2 - 1 > 0 \Leftrightarrow \log_2(x^2-1) = 0 \wedge x^2 > 1 \Leftrightarrow x^2 - 1 = 1 \wedge (x < -1 \vee x > 1) \Leftrightarrow$

$\Leftrightarrow x^2 = 2 \wedge (x < -1 \vee x > 1) \Leftrightarrow (x = -\sqrt{2} \vee x = \sqrt{2}) \wedge (x < -1 \vee x > 1) \Leftrightarrow x = -\sqrt{2} \vee x = \sqrt{2}$

$B: |\sqrt{2}x| \leq \sqrt{6} \Leftrightarrow \sqrt{2}x = -\sqrt{6} \vee \sqrt{2}x = \sqrt{6} \Leftrightarrow x = -\sqrt{3} \vee x = \sqrt{3}$

$B = [-\sqrt{3}, \sqrt{3}]$

$A \cap B = [1, \sqrt{2}[\cup]\sqrt{2}, \sqrt{3}]$

b) $f_r(A \cap B) = \{1, \sqrt{2}, \sqrt{3}\}$

$\overline{A \cap B} = [1, \sqrt{3}] \neq A \cap B$, logo $A \cap B$ não é fechado

(5.)

a) D:

x	$-\infty$	$-\sqrt{2}$		-1		0		$\sqrt{2}$	$+\infty$
$\sqrt{2-x^2}$	SS	0	+	+	+	+	+	0	SS
$\log(x+1)$	SS	SS	SS	SS	−	0	+	+	+
$\operatorname{sen}(x)$	−	−	−	−	−	0	+	+	+
T	SS	SS	SS	SS	+	SS	+	0	SS

$2 - x^2 \geq 0 \Leftrightarrow x^2 \leq 2 \Leftrightarrow x \geq -\sqrt{2} \wedge x \leq \sqrt{2}$

$x + 1 > 0 \Leftrightarrow x > -1$ $\quad D =]-1, 0[\cup]0, \sqrt{2}]$

$\operatorname{sen}(x) = 0 \Leftrightarrow x = k\pi, k \in \mathbb{Z}$

b) $A = \mathbb{N}$ $\quad A \cup D = [-1, 0[\cup]0, \sqrt{2}] \cup \mathbb{N}$ $\quad (A \cup D)' = [-1, \sqrt{2}]$

$\operatorname{int}(A \cup D) =]-1, 0[\cup]0, \sqrt{2}[$, logo não é aberto

$\overline{A \cup D} = [-1, 0[\cup]0, \sqrt{2}]$, logo é fechado

(6.) D:

x	$-\infty$	-2		0		2	$+\infty$
$\sqrt{4-x^2}$	SS	0	+	+	+	0	SS
x	−	−	−	SS	+	+	+
T	SS	0	−	SS	+	0	SS

$D = [-2, 2] \setminus \{0\}$

$4 - x^2 \geq 0 \Leftrightarrow x^2 \leq 4 \Leftrightarrow x \geq -2 \wedge x \leq 2$ $\quad \operatorname{int}(D \cup F) =]-2, 0[\cup]0, 2[$

$F = x = \{3, \frac{5}{2}, \frac{7}{3}, \frac{9}{4}, \frac{11}{5}, \frac{13}{6}, \ldots, 2\}$ $\quad f_r(D \cup F) = \{-2, 0, 2\} \cup F$

(7.)

a)

x	$-\infty$	$-\sqrt{5}$		$-\sqrt{3}$		$\sqrt{3}$		2		$\sqrt{5}$	$+\infty$
$(3-x^2)(5-x^2)$	+	0	−	0	+	0	−	−	−	0	+
$\lvert x-2 \rvert$	+	+	+	+	+	+	+	0	+	+	+
T	+	0	−	0	+	0	−	SS	−	0	+

$(3 - x^2)(5 - x^2) = 0 \Leftrightarrow 3 - x^2 = 0 \vee 5 - x^2 = 0 \Leftrightarrow x = -\sqrt{3} \vee x = \sqrt{3} \vee x = -\sqrt{5} \vee x = \sqrt{5}$

$x - 2 = 0 \Leftrightarrow x = 2$

$D = [-\sqrt{5}, -\sqrt{3}] \cup [\sqrt{3}, 2[\cup]2, \sqrt{5}]$

b) $\text{int}(D) =]-\sqrt{5}, -\sqrt{3}[\cup]\sqrt{3}, 2[\cup]2, \sqrt{5}[$

$fr(D) = \{-\sqrt{5}, -\sqrt{3}, \sqrt{3}, 2, \sqrt{5}\}$

$D' = [-\sqrt{5}, -\sqrt{3}] \cup [\sqrt{3}, \sqrt{5}]$

8. D:

x	$-\infty$	-2		$-\sqrt{3}$		0		$\sqrt{3}$		2	$+\infty$
x	$-$	$-$	$-$	$-$	$-$	0	$+$	$+$	$+$	$+$	$+$
$\log(4-x^2)$	$\exists\!\!\!/$	$\exists\!\!\!/$	$-$	0	$+$	$+$	$+$	0	$-$	$\exists\!\!\!/$	$\exists\!\!\!/$
T	$\exists\!\!\!/$	$\exists\!\!\!/$	$+$	$\exists\!\!\!/$	$-$	0	$+$	$\exists\!\!\!/$	$-$	$\exists\!\!\!/$	$\exists\!\!\!/$

$4-x^2 > 0 \Leftrightarrow x^2 < 4 \Leftrightarrow x > -2 \wedge x < 2$

$\log(4-x^2) \neq 0 \Leftrightarrow 4-x^2 \neq 1 \Leftrightarrow x^2 \neq 3 \Leftrightarrow x \neq -\sqrt{3} \vee x \neq \sqrt{3}$

$D =]-2, -\sqrt{3}[\cup]-\sqrt{3}, \sqrt{3}[\cup]\sqrt{3}, 2[$

B: $\begin{cases} 2 - \frac{1}{n} & \text{se } n \text{ impar} \\ 2 + \frac{1}{n} & \text{se } n \text{ par} \end{cases}$ $\quad B = \{1, \frac{5}{2}, \frac{5}{3}, \frac{9}{4}, \dots 2\}$

$\text{int}(D \cup B) =]-2, -\sqrt{3}[\cup]-\sqrt{3}, \sqrt{3}[\cup]\sqrt{3}, 2[$

$fr(D \cup B) = \{-2, -\sqrt{3}, \sqrt{3}, 2\} \cup B \setminus D$

9.

a)

x	$-\infty$	-1		0		1	$+\infty$
x	$-$	$-$	$-$	0	$+$	$+$	$+$
$1-\lvert x\rvert$	$-$	0	$+$	$+$	$+$	0	$-$	
T	$+$	$\exists\!\!\!/$	$-$	0	$+$	$\exists\!\!\!/$	$-$

$A =]-\infty, -1[\cup]0, 1[$

$1-|x| = 0 \Leftrightarrow |x| = 1 \Leftrightarrow x = -1 \vee x = 1$

b) $B = \{2, \frac{4}{3}, \frac{18}{9}, \dots 0\}$

majorantes de $A \cup B$: $x = [2, +\infty[$

$(A \cup B)' =]-\infty, -1] \cup [0, 1]$

(10.)

a) A:

x	$-\infty$		0		1		4	$+\infty$
$\log(x)$	SS		SS	$-$	0	$+$	$+$	$+$
$x-4$	$-$		$-$	$-$	$-$		0	$+$
T	SS		SS	$+$	0	$-$	SS	$+$

$A =]0,1[\cup]4,+\infty[$

$\log(x) = 0 \Leftrightarrow x = 1$

$x - 4 = 0 \Leftrightarrow x = 4$

B:

x	$-\infty$		$-\frac{\sqrt{2}}{2}$		0		$\frac{\sqrt{2}}{2}$	$+\infty$
$\sqrt[4]{e^2 - e^{6x^2-1}}$	SS		0	$+$	$+$	$+$	0	SS
$-x^3+2x^2-4x$	$+$		$+$	$+$	0	$-$	$-$	$-$
T	SS		0	$+$	SS	$-$	0	SS

$B = [-\frac{\sqrt{2}}{2}, 0[\cup]0, \frac{\sqrt{2}}{2}]$

$e^2 - e^{6x^2-1} \geq 0 \Leftrightarrow e^2 \geq e^{6x^2-1} \Leftrightarrow 2 \geq 6x^2 - 1 \Leftrightarrow 6x^2 \leq 3 \Leftrightarrow x^2 \leq \frac{1}{2} \Leftrightarrow x \geq -\frac{\sqrt{2}}{2} \wedge x \leq \frac{\sqrt{2}}{2}$

$-x^3 + 2x^2 - 4x = 0 \Leftrightarrow x(-x^2 + 2x - 4) = 0 \Leftrightarrow x = 0 \vee x = \frac{-2 \pm \sqrt{4-16}}{-2} \Leftrightarrow$

$\Leftrightarrow x = 0$

b) $A \cup B =]-\frac{\sqrt{2}}{2}, 0[\cup]0,1[\cup]4,+\infty[$

$int(A \cup B) =]-\frac{\sqrt{2}}{2}, 0[\cup]0,1[\cup]4,+\infty[$

minorantes de $A \cup B$: $x =]-\infty, -\frac{\sqrt{2}}{2}]$

$B' = [-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}]$

c) Falsa

(11.)

a)

x	$-\infty$		1		2		3	$+\infty$
$x-1$	$-$		0	$+$	$+$	$+$	$+$	$+$
$(x-2)(x-3)$	$+$		$+$	$+$	0	$-$	0	$+$
T	$-$		0	$+$	SS	$-$	SS	$+$

$A =]-\infty, 1[\cup]2,3[$

$x - 1 \geq 0 \Leftrightarrow x \geq 1$

$(x-2)(x-3) = 0 \Leftrightarrow x - 2 = 0 \vee x - 3 = 0 \Leftrightarrow x = 2 \vee x = 3$

b) $B = \{0, \frac{1}{2}, \frac{2}{3}, \dots, 1\}$

$fr(A \cup B) = \{1, 2, 3\} \cup B \setminus A$

c) $B \cup \{2\} \neq int(B \cup \{2\}) = \emptyset$, logo $B \cup \{2\}$ não é aberto

$B \cup \{2\} \neq \overline{B \cup \{2\}}$, logo $B \cup \{2\}$ não é fechado $(B \to 1)$

d)

i) V ii) V iii) F